Comparison of Ordinary Kriging and Multiple Indicator Kriging Estimates of Asuadai Deposit at Adansi Gold Ghana Limited*

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Abstract

Adansi Gold Ghana Ltd has taken over from the then Resolute Amansie Ltd and has three main historical deposits: Nkran, Adubia, Abore, and a new deposit, Asuadai. The company now wants to evaluate the resources at Asuadai which is about 7.5 km from Nkran deposit where Ordinary Kriging (OK) method had been used for estimation. Results of the estimation showed a consistent underestimation of tonnage and grade. Linear estimation models such as OK often produce good estimates but may encounter problems estimating recoverable reserves in cases where the distribution of samples is highly skewed. Multiple Indicator Kriging (MIK) is one of the popular non-linear methods that can handle skewed distribution such as that for gold grades, and addresses some of the deficiencies of the linear methods. This paper compares the estimation results obtained by using MIK and OK for the Asuadai deposit and establishes that MIK presents a better estimate than the OK, and recommends that MIK be used to estimate the gold resources of the Asuadai deposit and other deposits with similar geological settings.

Keywords: Indicator Kriging, Ordinary Kriging, Variography, Gold, Outlier

1 Introduction

Adansi Gold Ghana Ltd has taken over from the then Resolute Amansie Ltd and has three main historical deposits: Nkran, Adubia, Abore, and a new deposit, Asuadai. The former company used Ordinary Kriging (OK) for the resource estimation of the Nkran deposit which sits along strike to the Asuadai project area with similar geological settings, but had unsatisfactory grade and tonnage reconciliation (75%) during mining. The grade and tonnage have consistently been underestimated. The Nkran deposit is about 7.5 km south of Asuadai, as shown in Fig. 1. Thus, Adansi Gold Ghana Ltd finds it imperative to review this method and improve upon reconciliation to beyond 90%. Adansi Gold Ghana Ltd has been undertaking further exploration on the Asuadai deposit since it took over the company in order to refine the geological and resource models. Statistical analysis of the data indicates very highly skewed distribution with some outliers which might be causing excessive smoothing. Several methods such as indicator methods and conditional simulation are acclaimed (Spiers, 2011) to produce better estimates than Ordinary Kriging in handling such data. This research seeks to verify the applicability of Multiple Indicator Kriging (MIK), and whether it can improve upon reconciliation during mining.

2 Materials and Methods Used

Drill hole data obtained for the study were largely in soft copy, comma separated format, collected from the company's database section. The data included the collars, lithology, down-hole survey and assay. Other data acquired for the study were topography and density information. The data was validated and subjected to statistical analysis. This was followed by structural analysis (variography) for both OK and MIK using the GEOVIA Surpac software, version 6.5. Block modelling of the deposit was done after which the mineral resource was estimated by both OK and MIK using the aforementioned software. The study also gathered information from the mine on the geology of the deposit.

2.1 Geology of Asuadai

Hellman and Schofield (2011) and Asare *et al*., (2012) describe the Asuadai deposit as hosted in sheared interbedded metasedimentary rocks (greywackes and phyllites) that are intruded by numerous small felsic dykes with widths ranging between 0.5 m and 20 m. The deposit itself appears to be controlled by a NE-SW trending shear zone. The quartz veins containing gold mineralisation occur as vertically stacked conjugate vein sets normal to the trend of the shear zone. The zone varies in width from 5 to 50 m where mineralisation extends into the wall rocks. Most of the mineralisation occurs within this structural corridor. In general, the quartz veins and associated mineralisation trend 035° and dip between 30 and 60° to the NW. The quartz veins or flats are between 1 and 3 m wide.

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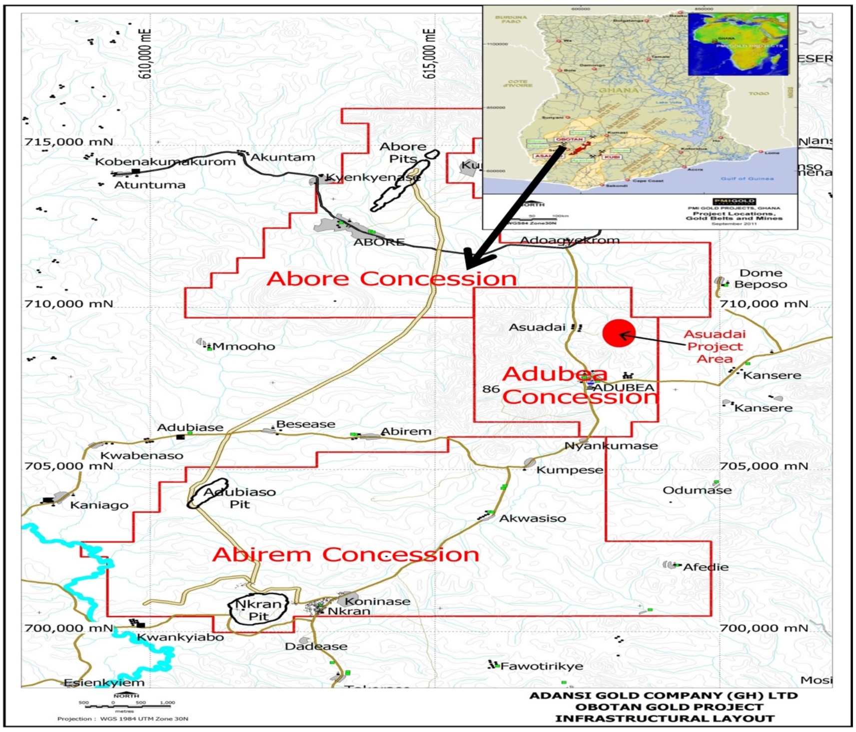

Fig. 1 Location of Asuadai Deposit

2.2 Data Management

2.2.1 Data Acquisition

Data obtained from the database section of the company included collars, lithology, down-hole survey, topography and density. In all, 136 holes (59 diamond drill holes and 77 reverse circulation holes) totaling 13 105.92 m, over a strike length of 650 m with depth up to 200 m below surface, were used. Fig. 2 shows the drill hole layout in plan. The holes were drilled on a grid of 25 m by 25 m with occasional infill of 12.5 m. Early reverse circulation holes were sampled by conventional splitting using the one stage riffle splitter at 2 m composite after which mineralised corridors were re-split to 1 m for re-assay. Split half-cores were bagged in lengths generally between 0.5 and 2 meters for gold fire assay analysis. In low grade intersections, the sample interval was between 2 and 3 meters. However, this was reduced to between 0.5 m and 1 m lengths in higher grade areas mainly controlled by lithology, alteration and mineralisation.

2.2.2 Data Validation

The drill-hole database was validated for items such as missing information, illogical data entries and others such as sample interval overlaps. The data validation work revealed no significant errors other than gaps in the sampling intervals due to selective sampling of the diamond drill holes.

2.3 Statistical Analysis

2.3.1 Combination of DD and RC Samples

A relation exists between sample support and the shape of the distribution of samples. Increasing the support has effect of reducing the dispersion variance due to 'smoothing effect' (Al-Hassan, 2011). Due to this relation, samples of identical support should be used otherwise the statistics, apart from the mean, would be meaningless (Isaaks and Srivastava, 1989). If groups of samples are to be combined for statistical analysis, they should belong to statistically similar distributions (Annels, 1991). To investigate whether the samples produced by DD and RC drilling belonged to similar populations, F- and t- tests were carried out on these two populations.

The F-test was used to determine whether the DD and the RC sample data came from populations with identical variances at 5% level of significance (Davis, 1986). The results of the test are shown in Table 1.

Table 1 F-test Results for the Asuadai Deposit

Deposit	Calculated Value(F)	Tabulated Value (F_α)
Asuadai	0.78	1.05

The calculated value, $F < F\alpha$, hence H_0 was accepted. Thus the two sample distributions were deemed identical. Having proven from the F-test that the variances of the two sample types were not significantly different, the equivalence of their means could be tested using the t-test (Davis, 1986). The results of the t-test at 5% level of significance are shown in Table 2.

Table 2 t-test Results for the Asuadai deposit

Deposit	Calculated Value(t)	Tabulated Value (t_α)
Asuadai	2.24	3.71

The results of the test also indicated that there was no significant difference between the means of the distributions and that they came from identical populations. Hence the two samples were combined for further statistical analysis and resource estimation.

2.3.2 Distribution Analysis

Selection of data was done, using geological and mineralogical domains. The data for the Asuadai resource analysis for the Ordinary Kriging and MIK were taken from 2 m composites. Histogram and probability plots were generated to determine types of distributions and identify outliers. Summary statistics for the 2-m composites are shown in Table 3 whilst Fig. 3 shows the histogram with cumulative frequency curve. The gold mineralisation is characterised by a positively skewed distribution as is expected for gold deposits (Davis, 1986).

Table 3 Summary Univariate Statistics of 2-meter Gold Composites

Number of samples	4940
Minimum value (g/t)	0.001
Maximum value (g/t)	59
Mean (g/t)	0.4
Median (g/t)	0.05
Variance	1.99
Standard Deviation	1.41
Coefficient of variation (CV)	3.53
Skewness	18.17
Kurtosis	583.02

From the summary statistics (shown in Table 3), the coefficient of variation of 3.53 indicates moderate-to-high variation for the gold deposit (Vann *et al*., 2000). It is an indication that few outliers may exist and estimation problems may be expected.

2.3.3 Outlier Analysis (Upper Cuts)

Upper cuts are often applied to highly variable deposits such as gold to limit the disproportionate influence of a few high-grade outlying samples if they are not identified and appropriately dealt with (Hellman & Schofield, 2011). The histogram shows a positively skewed distribution. The log probability graph (Fig. 4) shows several distinct sub-populations, with thresholds that show as inflexures (Rossi and Deutsch, 2014), most notably at 0.5, 2.0 and 10 ppm Au. The Asuadai deposit displays few extreme high-grade values above 10 g/t. Thus, an upper cut-off grade of 10 g/t was applied to the data prior to estimation by reducing all grades above 10 g/t to 10 g/t. Eleven outlier samples were detected.

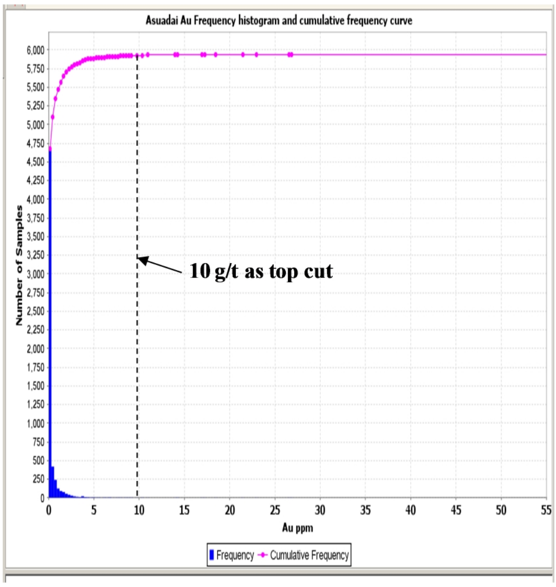

Fig. 3 Histogram/Cumulative Frequency of 2 m Composites

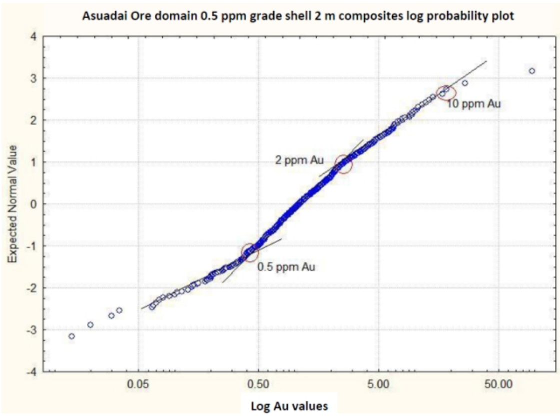

Fig. 4 Asuadai Log Probability Plot of the Distribution of 2 m Composites

2.4 Resource Estimation

2.4.1 Variography for OK

An omni-directional semi-variogram along azimuth of 00°, Plunge of 00°, spread of 90°, and lag spacing of 2 m was calculated and modelled (Fig. 5). This enhanced the estimation of the nugget variance. Directional semi-variograms were thereafter calculated and modelled with lag spacing of 10 m, spread of 15° and angular tolerance of 30° (to allow for adequate data capture) and a maximum lag distance of 150 m along strike, across strike and downhole. In all the cases they indicated two nested structures. Figs. 6 and 7 show the semi-variograms along strike (which was the direction of maximum continuity) and across strike respectively. The directional semi-variogram model parameters are shown in Table 4.

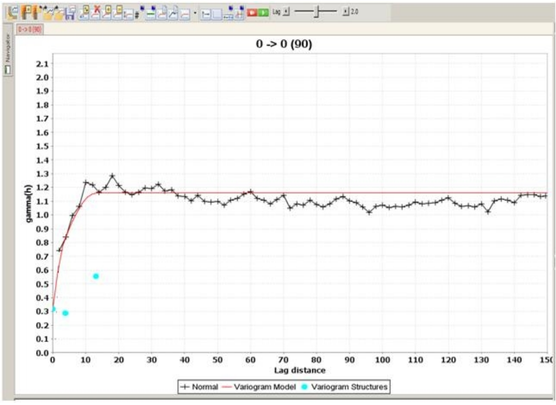

Fig. 5 Omni-directional Variogram (Downhole)

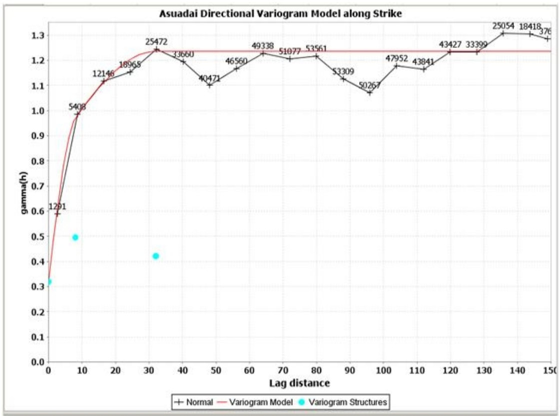

Fig. 6 Directional Semi-variogram Along Strike

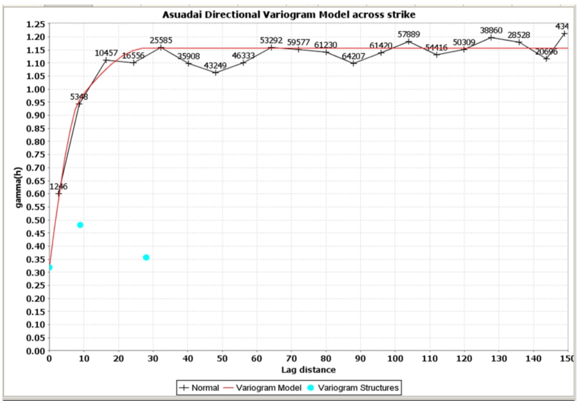

Fig. 7 Directional Semi-variogram Across Strike

Table 4 Semi-variogram Parameters for 2-m Sample Values

Down-hole	Across strike	Along strike	Direction
Spherical			Model
0.32	0.32	0.32	C_0
0.41	0.48	0.49	C_1
0.44	0.36	0.42	C_2
3.08	5.07	8.01	a_1
11.69	20.28	32.04	a_2

The nested structure is a result of alternation of the main intrusive rock controlling mineralisation that is continuous, exhibiting the long range structures; and the wall rocks (waste to weak mineralisation) being xenoliths within the intrusive that have shorter length, exhibiting the short range structures. Thus the long range anisotropic ratios were used for the resource estimation. This is consistent with the wireframe model of the gold mineralisation and known structural controls in the Asuadai area. The search ellipsoid has its major, intermediate and minor radii oriented along the strike, across strike and down dip respectively.

2.4.2 Variography for MIK

For the MIK, eight cut-offs were used: 0.5, 1, 1.5, 2, 2.5, 3, 3.5 and 4 g/t. This was done at a bin size of 0.5 using the Basic Statistics Tool within Surpac. Table 5 shows summaries of the conditional statistics. The sample values within the class were transformed into 0 and 1 from which directional indicator variograms were calculated and modelled for each cut-off. The indicator is obtained as (Hohn, 1988):

$$i(x;Z_c) = \begin{Bmatrix} 1, \text{if } Z(x) \le Z_c \\ 0, \text{if } Z(x) > Z_c \end{Bmatrix} \quad (1)$$

where, i is the indicator variable defined at location x for the cut-off Z_c, and $Z(x)$ is the grade at location x.

A lag spacing of 10 m, spread of 15° with an angular tolerance of 30° and a maximum lag distance of 150 m were used. The directional experimental variograms were calculated and modelled along strike, across strike and downhole. All depicted nested structures of spherical models. Samples of such model for 0.5 cut-off are shown in Figs. 8 to 10, and the parameters are detailed in Table 6.

Table 5 Conditional Statistics

Threshold	Class count	Class mean	Class median	Class Frequency
<0.5	4999	0.076	0.067	0.842
0.5 - 1	389	0.698	0.665	0.064
1 - 1.5	177	1.221	1.191	0.03
1.5 - 2	118	1.718	1.682	0.02
2 - 2.5	71	2.217	2.16	0.012
2.5 - 3	44	2.697	2.634	0.007
3 - 3.5	28	3.205	3.122	0.005
3.5 - 4	33	3.740	3.651	0.006
4 ≥	81	8.480	5.820	0.014
				1

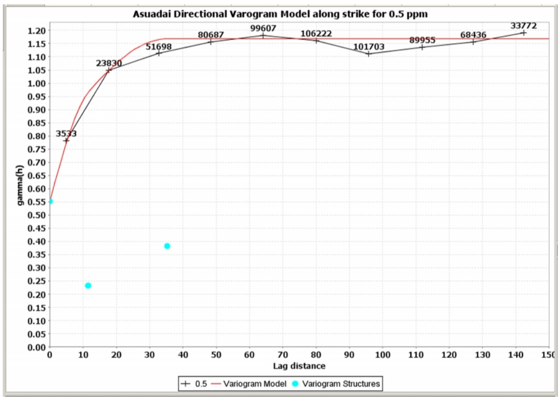

Fig. 8 Indicator Semi-variogram along Strike for 0.5 Cut-off

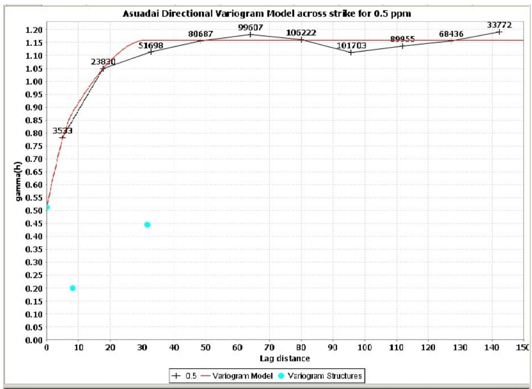

Fig. 9 Indicator Semi-variogram across Strike for 0.5 Cut-off

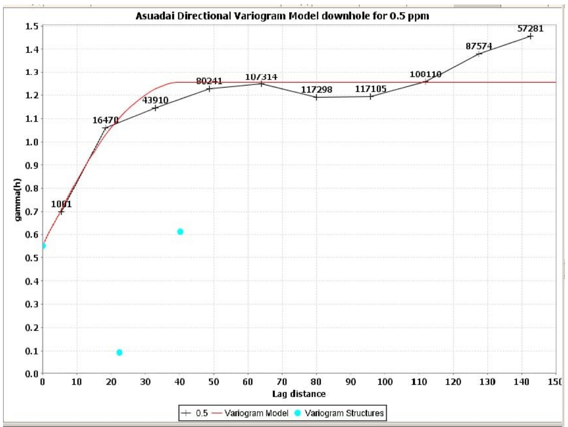

Fig. 10 Indicator Semi-variogram Downhole for 0.5 Cut-off

Table 6 Indicator Semi-variogram Parameters for Cut-off of 0.5

Down-hole	Across strike	Along strike	Direction
0.5			Cut-off
Spherical			Model
0.55	0.55	0.55	C_0
0.12	0.18	0.23	C_1
0.57	0.44	0.34	C_2
8.31	10.38	11.62	a_1
25.18	31.47	35.25	a_2

2.5 Block Modelling

A block model was generated from the orebody wireframe model using the Block model tool in Surpac. The block modelling parameters are presented in Table 7. The parent cell size of 15 x 3 x 3 m corresponds to approximately one half the average drill hole spacing along strike, average mining width of the deposit across strike and the likely mining bench height (3 m) in the vertical dimension respectively. Data search criteria employed took into account clustering of the local data, the geometry and continuity of local grade. All blocks were domained by geology, weathering type and given unique density values for each domain based on rock type and/or weathering.

Table 7 Block Model Parameters

Type	Y (m)	X (m)	Z (m)
Minimum Coordinates	709075	617925	-5
Maximum Coordinates	709840	618351	325
User Block Size (parent)	15	3	3
Min. Block Size	7.5	1.5	1.5

2.6 Ordinary Kriging (OK)

Ordinary kriging was executed using Surpac and the attributes of the search ellipsoid are shown in Table 8. Minimum and maximum number of samples used per block are 2 and 32 respectively. The resource estimates at various cut-off grades are also shown in Table 9.

Table 8 Ellipsoid Orientation and Kriging Parameters

Ellipsoid orientation			Kriging parameters			
Major radius	Major/ Intermediate	Major/Minor	Nugget	Sill	strike	Dip
32.04	1.58	2.74	0.31984	1.234	30	72

Table 9 OK Resource Estimates

Cut-off grade (g/t)	Volume $x10^6$ (m^3)	Tonnes $x\ 10^6$	Grade Au (ppm)	Gold $x10^6$ (oz)
0.5	1.113	1.810	1.2	0.069
1	0.676	1.701	1.41	0.062
1.5	0.218	0.951	1.94	0.025
2	0.074	0.324	2.51	0.010
2.5	0.025	0.042	3.12	0.004
3	0.011	0.020	3.57	0.002
3.5	0.004	0.007	4.14	0.001
4	0.002	0.003	4.95	0.000

2.6.1 OK Model Validation

OK block models were validated to assess the accuracy of grade. Cross sectional views of colour-coded drillhole composites were superimposed on similarly colour-coded block grades (Rossi and Deutsch, 2014). The best block model was the one whose colour codes compared closest to those of the sample grades. Fig. 11 shows one of such cross-sectional views of the OK block model along 709300 N.

2.7 Resource Estimation by MIK

For the kriged estimates of the indicators, which represent the proportion of a block below a cut-off, and the means of the raw data within the cut-off group, the grade of the block is estimated. The results are shown in Table 10.

2.7.1 Validation of MIK Models

Like the case of OK, drill holes were superimposed on the Multiple Indicator Kriging block models and the best one was selected. Fig. 12 shows one of such cross-sections along 707300N with good grade correlation for the accepted model.

Table 10 MIK Resource Estimates

Cut-off grade (g/t)	Volume x 10^6 (m)	Tonnes x 10^6	Grade Au (ppm)	Gold x10^6 (oz)
0.5	0.813	1.967	1.790	0.113
1	0.798	1.931	1.810	0.109
1.5	0.564	1.365	2.000	0.085
2	0.222	0.537	2.450	0.041
2.5	0.064	0.155	3.100	0.016
3	0.028	0.068	3.800	0.008
3.5	0.006	0.015	4.510	0.002
4	0.002	0.005	5.220	0.001

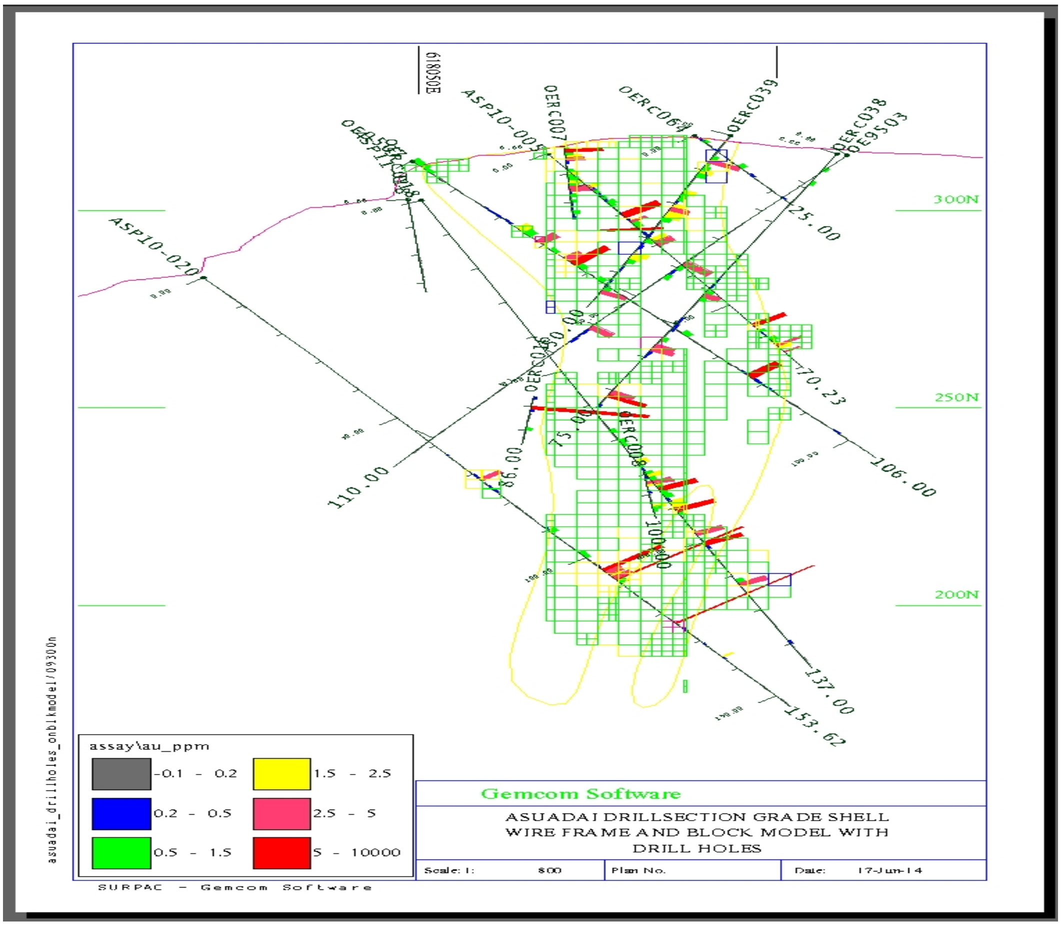

Fig. 11 Cross-sectional View of Validated OK Block Model along 709300 N

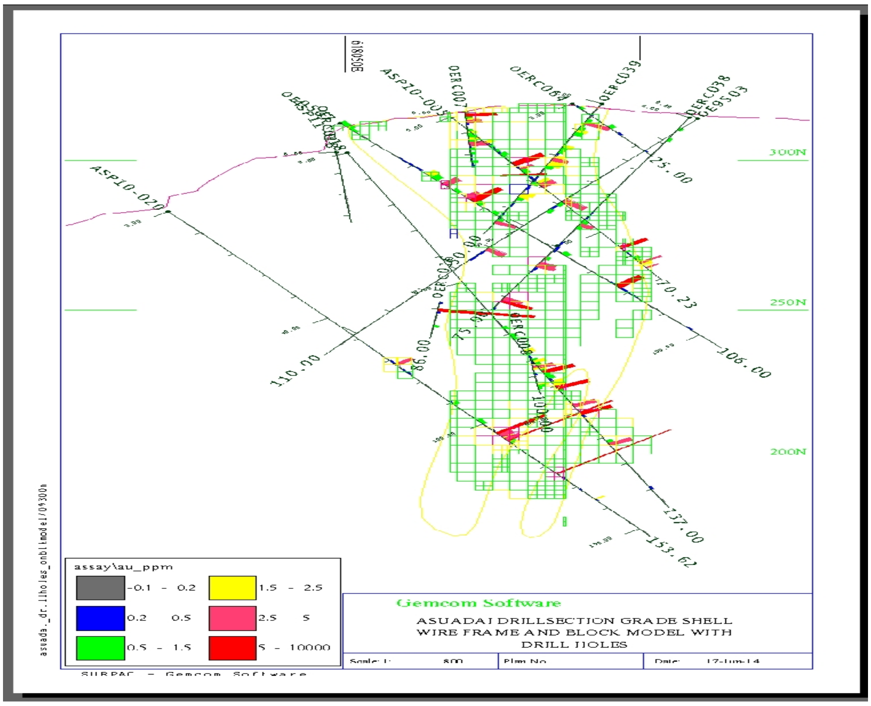

Fig. 12 Cross-sectional View of Drill Holes Superimposed on Ore Block Using Multiple Indicator Kriging along 709300N

3 Results and Discussion

A comparative statistics of the block grades for both estimates, shown in Table 11, indicates that there is more smoothing effect in the OK than the MIK estimates. This is verified from the relatively lower variance and CV of the OK estimates.

Table 11 Comparison of Block Model Estimates for OK and MIK

Statistics Statistics	OK	MIK
Minimum	0.01	0.01
Maximum	9.01	9.45
Median	1.1	1.34
Mean	0.36	0.37
SD	0.58	0.79
Variance	0.34	0.62
CV	1.61	2.13

Using the classification criteria of the company, the resources were classified at a cut-off of 0.5 ppm. For a measured resource, a drill sample density of between 20 and 25 m was chosen (based on spatial continuity and known geological continuity) and at least 20 samples for block estimate points. In the case of indicated resource, a sample density of 25 to 35 m between holes was chosen and 15 samples per block used, and for the inferred resource, a sample density of between 35 and 50 m between holes was applied with a minimum of 2 samples for block estimates (Gleeson and Naidoo, 2010). No measured value was obtained. Table 12 shows the resource categories (indicated and inferred) for both estimation methods at a cut-off of 0.5 ppm. The results show that the MIK presented higher estimates than the OK.

Table 12 Resource Classification at 0.5 ppm Cut-off for OK and IK

Method	Indicated			Inferred		
	Tonnes (millions)	Grade (g/t)	Oz (millions)	Tonnes (millions)	Grade (g/t)	Oz (millions)
OK	1.11	1.15	0.041	0.62	1.31	0.026
MIK	1.21	1.71	0.066	0.675	1.95	0.042

A comparison of the grade-tonnage curves for the global resource was also carried out as shown in Figure 13. Generally at any given cut-off, the OK in most cases provides lower tonnage and average grade than MIK. This is an indication of MIK's ability to handle recoverable tonnages in cases where the distribution of samples is highly skewed (Vann, 2007). The high skewness is attributable to some few high values. This was evidenced in the extensional drilling of the Nkran, which is 7.5 km south of Asuadai, where grades were encountered in the pit walls and waste during the resource upgrade drilling of the old pit and part of waste dump area. This may have accounted for the reconciliation challenges and consistent under-estimation of OK during their time of mining.

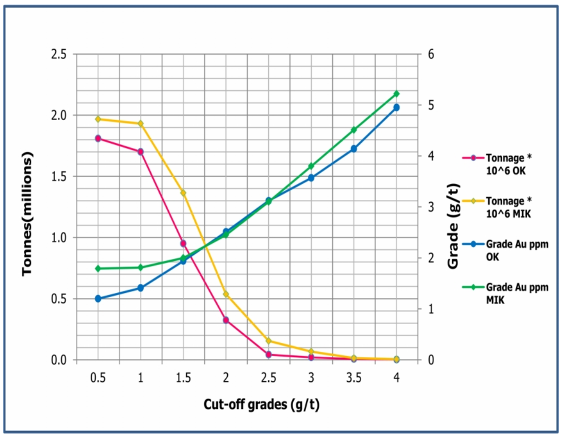

Fig. 13 Grade-Tonnage Curves of OK and MIK Block Estimates

4 Conclusions and Recommendations

4.1 Conclusions

(i) There is more smoothing effect in the OK than the MIK estimates as seen in the lower variance and CV values (Table 11); and

(ii) MIK estimates are closer to the unknown actuals than the OK estimates (Fig. 13).

4.2 Recommendations

Based on the findings and conclusions made in this study, it is recommended that:

(i) Care should be taken to control bias by effective top cutting when using OK or any other appropriate method since the deposit is nuggety and contains a few very important outliers; and

(ii) MIK may be used instead of OK in Asuadai and other deposits with similar geological settings in Adansi Gold.

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