

Rainstorm Intensity–Duration–Frequency Model for Tarkwa, Ghana*

¹D. Oti, ¹A. Asante-Annor, ²K. Bigson, ³G. Avane

¹University of Mines and Technology, Tarkwa

²WA Polytechnic, WA

³Goldfields Ghana Limited, Tarkwa

Oti, D., Asante-Annor, A., Bigson, K. and Avane, G. (2019), “Rainstorm Intensity–Duration–Frequency Model for Tarkwa, Ghana”, *Ghana Mining Journal*, Vol. 19, No. 1, pp. 21 - 28.

Abstract

Rainstorm intensity–duration–frequency relationship is a tool required for appropriate planning and designing of drainage systems. In this study, 22 years of annual peak daily rainstorm data distribution was tested to verify if it followed Lognormal, Gumbel, or Log Pearson Type III distribution. The distribution to which it belonged was used to build rainstorm intensity model of 0.5, 2, 5, 8, 10, 12, 15, 18, and 24 h durations. The findings revealed that the observed peak annual rainstorm followed Gumbel and Log Pearson Type III distributions at 0.05 significant level. Sherman rainstorm intensity model was also adopted and the parameters determined. Data generated with the Gumbel distribution function best fitted the Sherman model followed by those by log Pearson Type III. The rainstorm models developed with Sherman’s equation were recommended for use in the Tarkwa area.

Keywords: Flood, Gumbel Distribution, Water Resources, Rainstorm

1 Introduction

Rainstorm intensity–duration–frequency (IDF) evaluation of watersheds is important for determining the amount of rainstorm that could be expected to fall within a given duration in a watershed (Chow, 1978). The idea of IDF analysis is to enable capturing of peak direct runoffs that go through a given water channel of a watershed for various rainstorms over a duration. Also, to determine the frequency at which such incidence occur and re-occur. This thus enables engineers to determine accordingly, drainage sizes of a hydraulic structure that is required for a channel to withstand a peak flow during the life span of the constructing structure. Therefore, IDF curves provide vital information for drainage work designs such as culverts and storm drains.

Owing to its vitality for hydrologic engineering designs, IDF has been determined in many parts of the world (Elsebaie, 2012) with aims at improving flood protection, agricultural development, water resources and town planning and development (Canterford *et al.*, 1987; Zope *et al.*, 2016). But in Ghana, IDF has not yet been established for many watersheds including urban areas (Arhin-Acquah, 2016), although annual recurrence of floods from rainstorms has been reported in many of these areas. The fewer parts of the country’s urban watershed with established IDF include Kumasi (Arhin-Acquah, 2016) and Accra (Logah *et al.*, 2013). The flooding areas are characterized with undersized artificial storm drainage systems and

clogged natural drainage systems by solid municipal wastes (Rain *et al.*, 2011). To intervene the flood recurrence in Ghana therefore, is to establish IDF models nationwide and as well to urge storm water drainage designers and builders to adopt the models in their designs. This might cut down continual loss of lives and properties. Why to date the IDF has not been established nationwide, in Ghana, is unclear but the authors believes it is partly because of unavailability of short duration rainstorm data, that if were available would accurately capture the beginning, during, and end of rainstorm event in these watersheds.

The goal of this study therefore; is to determine an appropriate probability distribution model that best describes peak annual rainfall pattern and to construct IDF curves for the watersheds at Tarkwa, a vital gold mining town in Ghana. To achieve this goal; Lognormal, Gumbel and Log Pearson Type III (LPT III) distributions models were adopted and tested on the data set.

One of the benefits of availability of IDF curves is that, it is a decision making tool for civil engineering designs of hydraulic structures meant for flood protection against loss of personal properties and preservation of lives in floodplains. The establishment of the IDF relationship for the watersheds extends to as far as 100 years of return period. Another benefit is, when constructed and coupled with rational method, it enables peak runoff of any given rainstorm amount, its duration, and return periods to be soundly estimated (Gupta, 2001).

2 Resources and Methods Used

2.1 Study Area

Tarkwa is situated in the western region of Ghana and falls within south-western equatorial climatic zone, and is bounded by Huni River to the north, Bonsa River to the south and Ankobra River to the west, with both the Huni and Bonsa rivers serving as tributaries to the Ankobra River (Fig. 1). There are two wet seasons (March-July and September-November) (Fig.2) in the area; primarily influenced by moist south-west monsoon winds from the Atlantic Ocean and a dry dust-laden north-east trade winds from the Sahara desert. The mean annual rainstorm of the area is approximately 2000 mm with resulting flood recurrences (Rain *et al.*, 2011). Peak rainstorm trend study (not shown here), for the last two decades (1996 – 2016), is suggesting that maximum annual daily rainstorm at Tarkwa is on ascendancy; although it is not statistically significant at 0.05 significant level. However, the trend line indicates a tendency for increase of frequency of annual flood events. This finding has also been reported of other towns and cities in Ghana (Rain *et al.*, 2011).

The topography of the area comprises a series of prominent ridges and valleys (approximately 200 m

in height above sea level) within a generally a low lying area with a large low lying flood plain to the northwest. There are smaller streams that drain the area westward to the Huni River (Karpeta, 2000).

The vegetation comprises a mixture of tropical rain forest and semi-deciduous forest. Deforestation due to subsistence farming, mining activities, timber exploitation, firewood collection, charcoal production by the local population has further degraded the primary vegetation in the area to secondary forest, scrub and cleared land. The mean monthly temperature ranges from 24 °C to 30 °C, whilst relative humidity ranges from 70% to 90% (Akabzaa and Darimani, 2001). Actual evapotranspiration in the Tarkwa area is between 851-950 mm/y (Anayah *et al.*, 2013).

2.2 Data Source

Daily rainstorm data, starting from 1988 to 2016 (29 years) was acquired from Meteorological Services Office at University of Mines and Technology, Tarkwa, Ghana. Other acquired daily rainstorm data from Gold Fields Ghana Limited that spanned from 2005 to 2016 (12 years) was also used.

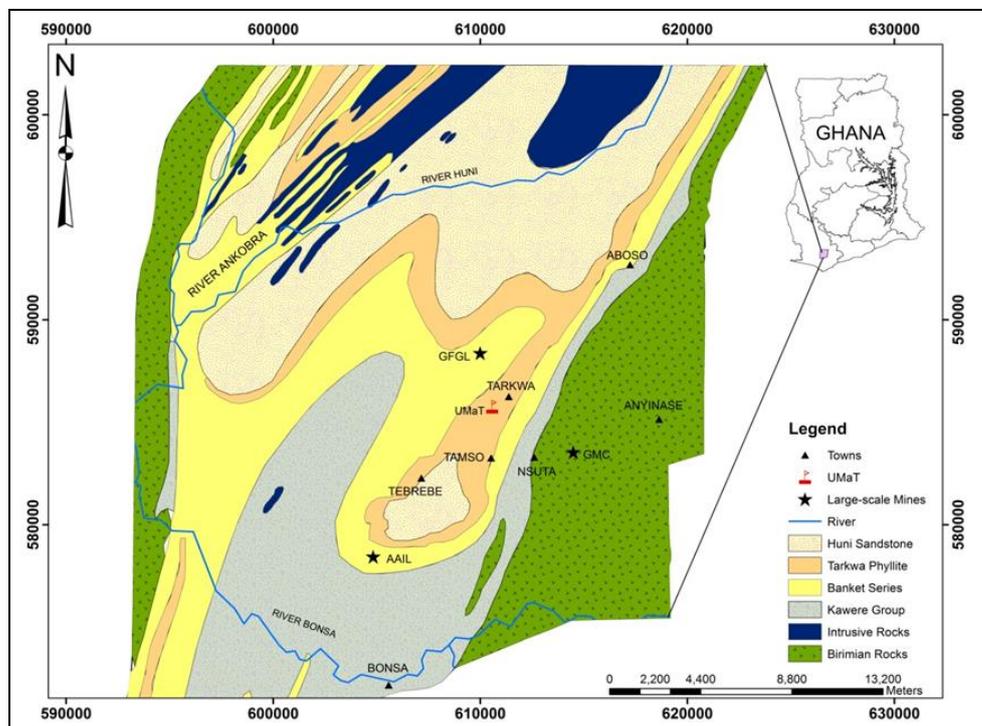


Fig. 1 Watersheds of Bonsa, Huni and Ankobra Rivers

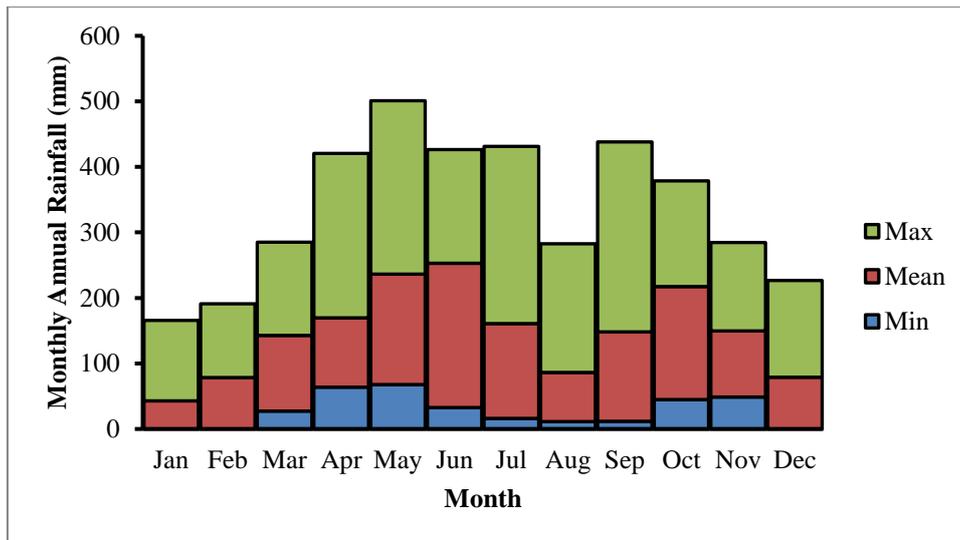


Fig. 2 Minimum, Mean and Maximum Monthly Rainstorm from 1988 to 2016

Of the 29 years daily rainstorm data from the Meteorological Services Office, 8 years of them; that ranges from 1988 to 1995; were considered unsuitable because of presence of significant missing data records. The rest of which had no missing records were considered suitable, and were subsequently used for the evaluation.

2.3 Development of IDF

To develop IDF requires shorter as much as longer duration rainstorm values, but the only observed rainstorm data readily available was the longer (daily) rainstorm records. Therefore to obtain the shorter duration rainstorm data, the authors adopted Indian Meteorological Department (IMD) empirical reduction formula. The IMD has been employed by others for shorter duration rainstorm value derivations from daily (24 h) recorded rainstorm data. Such authors include Bhatt *et al.* (1996), Chowdhury *et al.* (2007), and Logah *et al.* (2013). The generated data, with the IMD method, was considered as the “observed data” for the purpose of this study. Probability distribution of the observed data was then tested on Lognormal, Gumbel, and Log Pearson Type III (LPT III) distributions with an objective of identifying which of the distributions that the observed peak rainstorm data followed. In most cases Gumbel, and to a lesser extent (LPT III) are often the distribution functions that are adopted for analysis of rainstorm distributions. For instance, Bhatt *et al.* (1996), Logah *et al.* (2013), and Zope *et al.* (2016) all adopted the Gumbel distribution on assumption basis, while Garcia-Bartual and Schneider (2001) actually confirmed that the Gumbel was the best choice for rainstorm probability distribution studies. LPT III distribution and is even considered as the standard for frequency analysis of annual

maximum floods in the United States (Benson, 1968).

Each density function is associated with peculiar frequency factors which could be read either directly from factor tables or calculated with available mathematical formulae. To generate return periods and hence the probabilities of the observed rainstorm data for this study, Weibull method was adopted. This was to set the observed data sets up for the fitting into the lognormal and LPT III distribution models. However, for the Gumbel distribution assessment, reduced Gumbel variate was used.

2.3.1 Log Normal Distribution

Log normal distribution function is also usually adopted for data set distribution when a logarithm transformation of the observed data set results in a normal distribution. Generally, rainstorm depth, R_T (mm), for a given return period can be written as:

$$R_T = \mu(1 + K_T C_v) \quad (1)$$

where R_T is the candidate rainstorm values of a given return period and K_T is Chow’s log normal frequency factor.

To determine the Chow’s lognormal frequency factor (K_T) in the log normal probability distribution function, coefficient of variation (C_v) was first determined by dividing the standard deviation (σ) by the mean (μ) of the candidate peak rainstorm data. From log normal frequency table (Chow, 1978); lognormal frequency factor was determined, using the probability equal or greater than a given variate on the frequency table.

2.3.2 Gumbel Distribution

Gumbel distribution function is normally adopted for IDF analysis because of its suitability for extreme data analysis (Garcia-Bartual and Schneider, 2001). The method determines 2, 5, 10, 25, 50, and 100 years return period for each duration of rainstorm event. The rainstorm, R_T (mm), for each duration and for a given return period, T , is given as:

$$R_T = R_{av} + KS \quad (2)$$

where R_{av} is the average of candidate rainstorm values and K is Gumbel frequency factor is also given as:

$$K = \frac{\sqrt{6}}{\pi} \left[0.5772 + \ln \left[\ln \left[\frac{T}{T-1} \right] \right] \right] \quad (3)$$

S is standard deviation of the distribution of the candidate rainstorm values.

2.3.3 LPT III

Log Pearson Type III distribution function, as the name depicts, involves taking logarithm of the candidate rainstorm values. In this method the parameters, which are the mean and the standard deviation, were derived from log transformed rainstorm data. The distribution function can be expressed as:

$$R^L_T = R^L_{av} + K_L S^L \quad (4)$$

where R^L_T is logarithm of candidate rainstorm values at a given return period, R^L_{av} is logarithm of average of candidate rainstorm values at a given period, S^L is logarithm of standard deviation of the distribution of the candidate rainstorm values, K_L is Log Pearson frequency factor, can be determined from hydrologic table (Chow, 1978), when given a skewness coefficient of the distribution, C_s .

$$C_s = \frac{n \sum_1^n (R^L - R^L_{av})^3}{(n-1)(n-2)(S^L)^3} \quad (5)$$

where S^L is logarithm of standard deviation of the distribution of the candidate rainstorm values, R^L is logarithm of candidate rainstorm values.

2.4 IDF Equation Derivation

Three steps were followed to estimate parameters of IDF curves and were consistent with the steps described by Singh (1992); Wanielista and Youssef (1993); and Chow (1978). The first step involved identifying the probability distribution function that properly fitted the most severe storms data values

of the duration. The second step involved using the fitted probability distribution function to calculate the rainstorm intensities for each duration and for a set of return periods (e.g. 2, 5, 10, 20, 50, 100). The third step involved plotting of the IDF curve.

Adopting Sherman's IDF relationship, which is given as:

$$I_T = \frac{PT^\alpha}{t_d^\beta} \quad (6)$$

where I_T is rainstorm intensity (mm/h) at a given return period, P, α, β are constants, T is return period of the candidate rainstorm values, t_d is rainstorm duration. Also, where the frequency coefficient of the fitted model equals the numerator. For instance, if Gumbel were the chosen model, then its frequency factor, K would equal PT^α and a logarithm of (6) would be:

$$\log I_T = \log K - \beta \cdot \log t_d \quad (7)$$

A plot of logarithm of rainstorm intensity, $\log I_T$, as derived from modeled R_T as with Gumbel or R^L_T as with LPT III; against the logarithm of short duration rainstorm, $\log t_d$, would yield a straight line with β as a gradient and $\log K$ as an intercept. Furthermore, a plot of $\log K$ against $\log T$ would yield α as a gradient and P as an intercept.

2.5 Goodness of Curve Fit Test

Chi-square goodness-of-fit tests to analyze the degree of fitness of the observed data to the theoretical probability distributions were done. The null hypothesis of the tests being the observed data set fits a chosen probability distribution and is rejected if the critical value of the chi-square, for some degree of freedom and at some significant level, is less than the calculated value. The Chi-square goodness of fit function is given as:

$$\chi^2_{k-p-1} = \sum_k \left[\frac{(f_o - f_e)^2}{f_e} \right] \quad (8)$$

Where f_o is observed frequency, f_e is expected frequency, k is number of categories, p is number of parameters estimated.

After the derivation of the intensity models for the various return periods, the models were linearised and coefficient correlation between the observed data and models was used to test for goodness-of-fit of the models. F-tests were also done to verify if the intensity models were appropriate for the observed data sets.

3 Results and Discussion

3.1 Results

The fitted curves onto the observed maximum annual daily rainstorm can be seen in Fig. 3. The number of data categories here were the data sizes, and the parameters for both log normal and log Pearson Type III were two, while that for the Gumbel was three. The Chi-square goodness-of-fit tests revealed that at 0.05 significant level, no enough evidence exists to reject either Gumbel or LPT III distribution as a probability distribution that the maximum annual daily rainstorm data set fall into; but there is enough for the log normal distribution to be rejected (see Table 1). However, the rejection evidence diminishes at 0.005 significant level when the calculated chi-squared is less than the tabulated Chi-squared values.

The peak annual daily rainstorm depth as depicted by lognormal distribution model underestimates the daily rainstorm depth values while the LPT III

overestimates the peak rainstorm depth. In Table 2, the rainstorm values as produced by the Gumbel distribution model can be seen to consistently lie between these two values produced by lognormal and LPT III distribution models at various return periods. Additionally, the rainstorm depth increases with increasing return period for all the models. On the Gumbel frequency table, no data was available for one year return period. The tabulated data starts at the return period of two years.

In this paper, the Sherman's IDF relationship was adopted and has been employed by Elsebaie (2012) and Raghunath (2006) to determine rainstorm intensities. The derived values of the various parameters from the Sherman's equation can be seen in Table 3. Among the three distributions, Gumbel generated the highest coefficient, P , but the least α value; while lognormal generated the least, P , but the highest α value.

Table 1 Chi-Square Results at 0.05 Significant Level

	Degree of freedom	Table	Calc.	Decision
Log normal	19	30.144	37.265	Rejected
Gumbel	18	28.869	0.947	Not rejected
LPT III	19	30.144	0.430	Not rejected

Table 2 Peak Annual Daily Rainstorm Depth (mm) over the past 22 Years and the Frequency

Probability distribution	Return period (year)			
	1	10	50	100
Log Normal	45.28	136.40	175.00	190.30
Gumbel	N/A	137.17	176.81	193.57
LPT III	45.76	142.57	192.52	215.10

Table 3 Values of Various Parameters of Intensity Equations as Derived from Various Distributions

Probability distribution	P	α	β
Log Normal	21.37	0.28	0.67
Gumbel	29.67	0.19	0.67
LPT III	22.92	0.25	0.67

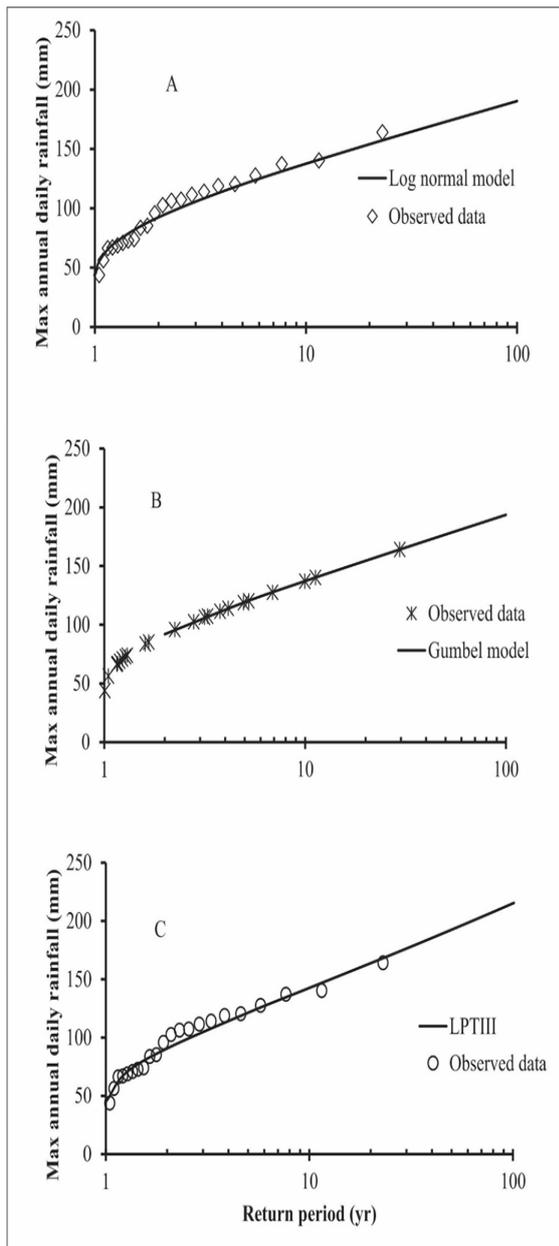


Fig. 3 Maximum Annual Daily Rainstorm Depth and their Return Periods

The P and α values varied slightly between the lognormal and LPT III distribution functions, but widely between Gumbel distribution function. On the other hand, β is the same for all the functions. This is consistent with the finding by Elsebaie (2012).

The coefficient of correlations between the models and that of the observed data, when linearized with logarithm functions, are listed in Table 4. The plots of the theoretical distribution functions and the generated data with IMD can also be seen in Fig. 4. Clearly, the IDF models for the LPT III fits well at return values of 50 and 100 years but do not fit well

at 1 and 5 years. This is again revealed in Table 4 where coefficient of correlations are 0.77 and 0.95 for the 1 and 5 years respectively; but 1.00 for both 50 and 100 years return periods.

For the lognormal models (Fig. 4A), those of the ends with 1, 5, and 100 years of return periods do not perfectly fit the observed data sets unlike the one in the middle with 20 years return period which is a perfect fit. For the Gumbel model however, all the models perfectly fit the observed data (Fig. 4B).

F - tests performed on the derived intensity model reveal that the p -values for all the models were less than 0.05 (Table 5) and therefore were all appropriate models for the rainstorm data.

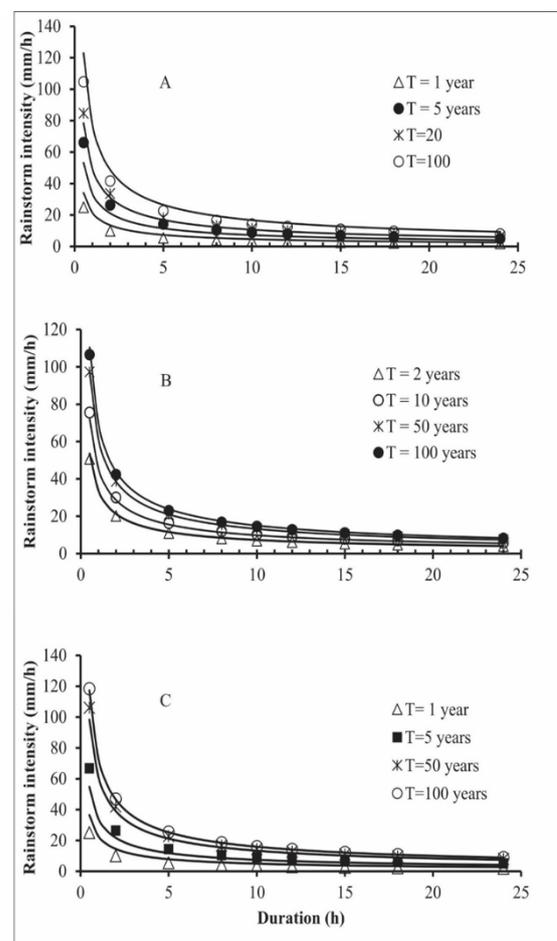


Fig. 4 Rainstorm Intensity – Duration Curves (A: Log Normal, B: Gumbel, C: LPT III)

Table 4 The Coefficient of Correlations between the Models and the Observed Data at Various Return Periods

Probability distribution	Intensity model (mm/h)	Return period, T (year)						
		1	2	5	10	20	50	100
Log normal	$I_T = \frac{21.37T^{0.28}}{t_d^{0.67}}$	0.84	0.94	N/A	N/A	1.00	1.00	0.99
Gumbel	$I_T = \frac{29.67T^{0.19}}{t_d^{0.67}}$	N/A	0.99	1.00	1.00	0.99	1.00	1.00
LPT III	$I_T = \frac{22.92T^{0.25}}{t_d^{0.67}}$	0.77	0.78	0.95	N/A	N/A	1.00	1.00

Table 5 The p-values of F-test of the Models on the Observed Data at Various Return Periods

Probability distribution	1	2	5	20	50	100
Log normal	4.3×10^{-3}	2.1×10^{-3}	N/A	1.03×10^{-7}	N/A	1.49×10^{-6}
Gumbel	N/A	1.5×10^{-7}	N/A	N/A	1.9×10^{-17}	1.59×10^{-9}
LPT III	1.2×10^{-2}	N/A	1.4×10^{-4}	N/A	4.4×10^{-4}	9.4×10^{-14}

3.2 Discussion

The main objective of this paper was to derive a model for rainstorm intensities for the Tarkwa watersheds. This was successfully done, but on a major assumption that the data generated with IMD formula represented the true observed short duration peak annual rainstorm data. This was based on the evidence that IMD has been successfully applied by others elsewhere. Another hypothesis was that, the observed data distribution follow either one, two or all of the tested theoretical distributions in this paper. In addition to this, it was also expected that once the theoretical distribution that the observed data followed had been identified, rainstorm intensity with as far as 100 year return period could be estimated, which is the extent that large hydraulic engineering constructions are designed to last.

The Chi-squared analysis of the data revealed that the observed rainstorm data follows Gumbel and Log Pearson Type III distributions at 0.05 significant level. It is enlightening to realize here that most often, Gumbel and to a lesser extent, LPT III are the most adopted distribution functions for rainstorm probability distribution analysis; although in the case of this study, log normal distribution as well can be employed, but at 0.005 significant level. In this study therefore, rainstorm intensities for the various distribution functions were successfully modeled (Table 4). These models tested with 1, 2, 5, 10, 20, 50, and 100 years of return periods of rainstorm were valid within 0.05 significant level for the Gumbel and LPT III only, otherwise it was valid for all the three distributions at 0.005 significant level.

The consistency of the derived β values of the rainstorm intensity models and those by Elsebaie suggests that β is robust and may be a characteristic of the rainstorm pertaining in the studied area. The shorter duration rainstorm values yielded higher rainstorm intensities than the longer durations. The rainstorms of shorter durations may also be responsible for such incidences as flash floods in the area. This is true when the intensity is higher than infiltration capacity and most of the storm water turns to be direct run-offs.

4 Conclusions

This study which highlights the Sherman's intensity models are valid for use in the Tarkwa watersheds and for their adoption for estimating peak annual rainstorm and return periods. From this study, the Gumbel distribution best fits the observed data followed by LPT III at 0.05 significant level and at various durations, but in general terms, the two distributions are close enough for either to be adopted when estimating the peak annual rainstorm at Tarkwa. The derived IDF curves are therefore recommended for use in predicting rainstorm intensities of the area, as well as for designing hydraulic structures for the Tarkwa area.

References

- Akabzaa, T. and Darimani, A. (2001), *Impact of mining sector investment in Ghana: A study of the Tarkwa mining region*, Third World Network, 70 pp.

Anayah, F.M., Kaluarachchi, J.J., Pavelic, P. and Smakhtin, V. (2013), "Predicting groundwater recharge in Ghana by estimating evapotranspiration", *Water International*, Vol. 38, No. 4, pp. 408-432

Arhin-Acquah, R. (2016), "Review of the Hydraulic Capacity of Some Existing Culverts on Selected Roads in Kumasi Metropolis (Ghana)", *Master's Thesis*, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana, 94 pp.

Benson, M. A. (1968), "Uniform flood-frequency estimating methods for federal agencies", *Water Resources Research*, Vol. 4, No. 5, pp. 891-908.

Bhatt, V. K., Tiwari, A. K. and Sharma, A. K. (1996), "Probability models for prediction of annual maximum daily rainstorm of Datia", *Indian Journal Soil and Water Conservation*, Vol. 24, No. 1, pp. 25-27.

Canterford, R. P., Pescod, N. R., Pearce, H. J., Turner, L. H. and Atkinson, R. J. (1987), "Frequency analysis of Australian rainstorm data as used for flood analysis and design", In *Hydrologic Frequency Modeling*, Springer Netherlands, pp. 293-302.

Chow, V. T. (1978), "Stochastic modeling of watershed systems". *Advances in hydroscience*, Vol. 11, No. 1, pp. 1-93.

Chowdhury, R. K., Alam, M. J. B., Das, P. and Alam, M. A. (2007), "Short duration rainstorm estimation of Sylhet: IMD and USWB method", *Journal-Indian Waterworks Association*, Vol. 39, No. 4, pp. 285.

Elsebaie, I. H. (2012), "Developing rainstorm intensity-duration-frequency relationship for two regions in Saudi Arabia", *Journal of King Saud University-Engineering Sciences*, Vol. 24, No. 2, pp. 131-140.

Garcia-Bartual, R. and Schneider, M. (2001), "Estimating maximum expected short-duration rainstorm intensities from extreme convective storms", *Physics and Chemistry of the Earth, Part B: Hydrology, Oceans and Atmosphere*, Vol. 26, No. 9, pp. 675-681.

Gupta, R. S. (2001), *Hydrology and hydraulic systems*, Waveland Press, USA, 855 pp.

Karpeta, W. P. (2000), "A Review of the Geology, Mining and Exploration of the Tarkwa Mine Area", *Unpublished*, pp. 15-40.

Logah, F. Y., Kankam-Yeboah, K. and Bekoe, E. O. (2013), "Developing Short Duration Rainstorm Intensity Frequency Curves for Accra in Ghana", *International Journal of Latest Research in Engineering and Computing*, Vol. 1, No. 1, pp. 67-73.

Raghunath, H. M. (2006), *Hydrology: principles, analysis and design*, New Age International, New Delhi India, Wiley Eastern Limited, 476 pp.

Rain, D., Engstrom, R., Ludlow, C. and Antos, S. (2011), "Accra Ghana: A city vulnerable to flooding and drought-induced migration", *UN-Habitat (Ed.)*, pp. 1-21.

Singh, V. P. (1992), *Elementary hydrology*, Prentice-Hall, N. J., USA, 973 pp.

Wanielista, M. P. and Yousef, Y. A. (1993), *Stormwater management*, John Wiley and Sons, 199 pp.

Zope, P. E., Eldho, T. I. and Jothiprakash, V. (2016), "Development of Rainstorm Intensity Duration Frequency Curves for Mumbai City, India", *Journal of Water Resource and Protection*, Vol. 8, No. 7, pp. 756.

Authors



Douglas Oti is a lecturer at University of Mines and Technology, Tarkwa, Ghana. He holds degrees in MS from North Carolina Agricultural and Technical State University, Greensboro, USA; and PhD from University of South Florida, Tampa, Florida, USA. His research interest spans across Operations Research, Hydrology, and Artificial Intelligence in Gold Mining.



Asare Asante-Annor is a lecturer at the University of Mines and Technology, Tarkwa. He holds a BSc (Hons) in Geological Engineering from the Kwame Nkrumah University of Science and Technology, MSc in Environmental Resources Management and a PhD in Environmental Engineering from Brandenburg Technical University, Cottbus, Germany. His areas of research include Environmental Geology, Groundwater Geochemistry, Vulnerability of Water Resources and Waste Management.



Kate Bigson is a lecturer at Wa Polytechnic, Wa, Ghana. She holds degrees in MPhil and MEd from University of Education, Winneba, Ghana. Her research interests are in Food and Nutritional Contents of African Diets and Water Quality ingestion in Towns and Villages in Africa.



Godfred Avane is the Geology Manager at Gold Fields Ghana Ltd. He holds a BSc (Hons) in Geological Engineering from the Kwame Nkrumah University of Science and Technology and MSc Geological Engineering from the University of Mines and Technology. His research interest are Geotechnical Engineering and Mineral Exploration.